Is the Fragility of a Liquid Embedded in the Properties of Its Glass?

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When a liquid is cooled below its melting temperature, it usually crystallizes. However, if the quenching rate is fast enough, the system may remain in a disordered state, progressively losing its fluidity upon further cooling. When the time needed for the rearrangement of the local atomic structure reaches approximately 100 seconds, the system becomes "solid" for any practical purpose, and this defines the glass transition temperature $T_g$. Approaching this transition from the liquid side, different systems show qualitatively different temperature dependencies of the viscosity, and accordingly they have been classified by introducing the concept of "fragility." We report experimental observations that relate the microscopic properties of the glassy phase to the fragility. We find that the vibrational properties of the glass well below $T_g$ are correlated with the fragility value. Consequently, we extend the fragility concept to the glassy state and indicate how to determine the fragility uniquely from glass properties well below $T_g$.

When a liquid is cooled, the loss of kinetic energy leads to an ordering of the molecules, which then crystallize at the melting temperature $T_m$. However, if cooled fast enough through $T_m$, some materials (glass-forming materials) are capable of sustaining a metastable liquid state and, upon further cooling, of freezing into a disordered glassy state ($1\rightarrow 4$). The law that describes the change of the viscosity with the temperature approaching the glass transition temperature, $T_m$, is highly material-specific and has led to the classification of glass-forming materials according to the concept of "fragility" ($5, 6$). The kinetic fragility, $m$, is directly related to the slowing down of the dynamics: It is defined in terms of the shear viscosity $\eta$ as

$$m = \lim_{T \to T_g} \frac{d\log(\eta)}{d(T_m/T)}$$

Therefore, $m$ is an index of how fast the viscosity increases while approaching the structural arrest at $T_m$, the temperature at which the structural relaxation time $\tau_m \sim 100$ s. At this latter temperature, through the Maxwell relation $\eta = G_\tau \tau$, one finds a viscosity of $\eta \sim 10^{10}$ poise (more likely, $\eta \sim 10^4$ poise for molecular glasses), whereas $10^{-4}$ poise is the "infinite" temperature limit in basically any material. Consequently, fragility values typically range between $m = 17$ for "strong" systems (those that show an Arrhenius behavior) and $m \sim 150$ for "fragile" systems, where the high cooperativity of the diffusive dynamics induces a high (and $T$-dependent) apparent activation energy. One interesting element of this classification lies in the attempt to relate the temperature behavior of a macroscopic transport property close to $T_g$, to the microscopic interactions driving the dynamics of the system. It has been found, for example, that the value of the fragility is empirically related to the kind of interaction potential among the particles constituting the system. Prototypical examples of fragile liquids are those composed by units interacting via isotropic bonds, such as van der Waals-like molecular liquids. The strong glass-forming liquids, on the other hand, are those characterized by strong covalent directional bonds that form space-filling networks. $O$-terphenyl (m = 80) and SiO$_2$ (m = 20) are characteristic examples of a fragile and a strong liquid, respectively. Hydrogen-bonded systems, such as glycerol (m = 50), are often called "intermediate" between strong and fragile liquids. Kinetic fragility has been found to be correlated to other properties of the glass-forming liquids, such as (i) the slope at $T_g$ of the temperature dependence of the configurational entropy (often referred to as thermodynamic fragility) ($7$) or (for classes of systems sharing similar $T_g$) the specific heat jump at $T_g$ ($8, 9$); (ii) the ratio between the maximum and the minimum of the boson peak; that is, of the bump observed at terahertz frequencies in the Raman and neutron-scattering spectra of glass-forming materials ($10$) [although this finding is controversial ($11$)]; (iii) the degree of stretching in the non-exponential decay of the correlation functions in the liquid close to $T_g$ ($12$); (iv) the statistics of the minima in a potential energy landscape-based description ($13, 14$) of the diffusion process in supercooled liquids ($15\rightarrow 17$); and, more recently, (v) the temperature behavior of the shear elastic modulus in the supercooled liquid ($18$). In all these studies, the fragility has been always related to (or defined through) macroscopic properties characterizing the liquid side of the glass transition. Although there are attempts to relate the fragility to the anharmonicity of the "hot" glass ($19$), no connection has been found up to now between the...

References and Notes

11. The experimental image presented here was obtained from digital charge-coupled device camera recordings of the original negative films. It can grab image pixels from an area of 1536 × 1024 pixels at a spatial sampling rate of 0.013 mm/pixel. The displacement analysis was done in the negative image with DigitalMicrograph 3.6 (Gatan).
12. For $T_g^c$ and $T_g^h$ we only measured the spacing in the upper crystal because the contrast is more homogeneous in this part of the image. For $T_g^c$ we measured spacings from both the upper and lower crystals. The error bars correspond to the standard deviation determined from the statistical average.
15. We thank U. Salzberger for excellent TEM specimen preparation, R. Höschle for his help during the operation of the ARM1250, and C. T. Koch and G. Richter for helpful discussions with the image simulations.

29 July 2003; accepted 25 September 2003
value of $m$ and the physical properties of the low-temperature glassy phase.

We show that, starting from a determination of the nonergodicity factor in the low-temperature glass, it is possible to identify a parameter that controls how fast the nonergodicity factor decreases when the temperature is increased, and that turns out to be proportional to the fragility $m$. Through this, we establish a way to determine the fragility of a system in the glassy phase well below $T_g$, independent of the way in which viscosity changes with decreasing temperature from the liquid side. By exploiting the harmonic approximation of the low-temperature dynamics, it is found that this parameter only depends on the characteristics of the static disorder, which in turn controls the vibrational eigenmodes of the glass. This result demonstrates the existence of a deep link between the diffusive interbasin dynamics and the vibrational intrabasin dynamics in glass-forming systems.

Recent extensive inelastic x-ray scattering (IXS) measurements of the dynamic structure factor have allowed a sizeable library of high-frequency (terahertz) dynamical properties of glasses to be constituted. Of interest here is the fact that the IXS measurements allow for the determination of the nonergodicity factor $f(Q,T)$, with a reliability that was not achievable via light or neutron scattering (20). The nonergodicity factor is the long time limit of the density correlator $\Phi_f(t)$; that is, the density-density correlation function $F(Q,t)$, normalized to the static structure factor $S(Q)$; $\Phi_f(t) = F(Q,t)/S(Q)$. The quantity $1 - f(Q,T)$ represents the amount of decorrelation introduced by the vibrational dynamics, and it depends on both the ($T$-dependent) amplitude of the vibrations and the degree of disorder of the glassy structure. In a low-temperature glass, $F(Q,t)$, apart from the Debye-Waller factor $\exp[-W(Q)]$, can be expressed as the sum of a constant term $S_{\text{el}}(Q)$, which represents the static structure factor of the atomic equilibrium positions (inherent structure), plus a time-dependent one, $F_{\text{inel}}(Q,t)$, which is the contribution of the atomic vibration around such equilibrium positions, a quantity that vanishes in the long time limit

$$F(Q,T) = e^{-\omega(Q)|t|}[S_{\text{el}}(Q) + F_{\text{inel}}(Q,t)].$$

(1)

Therefore,

$$f(Q,T) = \lim_{t \to \infty} \Phi_f(t) = \lim_{t \to \infty} \frac{S_{\text{el}}(Q) + F_{\text{inel}}(Q,t)}{S_{\text{el}}(Q) + F_{\text{inel}}(Q,t)} = \frac{1}{1 + S_{\text{el}}(Q)/S_{\text{el}}(Q)}$$

(2)

where we have defined $S_{\text{el}}(Q) = F_{\text{inel}}(Q,t) = 0$. $S(Q,\omega)$ is the Fourier transform of $F(Q,t)$ and is the quantity directly accessible in scattering experiments. From Eq. 1 it can be expressed as

$$S(Q,\omega) = e^{-\omega(Q)|t|}[S_{\text{el}}(Q)\delta(\omega) + S_{\text{el}}(Q,\omega)].$$

(3)

From an experimental point of view and according to Eqs. 2 and 3, the nonergodicity factor is derived from the ratio of the elastic to the inelastic scattered intensity, obtained from inelastic scattering measurements of the dynamic structure factor $S(Q,\omega)$ (21). A sense of the $T$ dependence of $f(Q,T)$ can be obtained from Fig. 1. Here, as an example, we report the IXS spectra at fixed exchanged wave vector ($Q = 2$ nm$^{-1}$) at different temperatures in glycerol. The inelastic (dashed lines) and elastic (dotted

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**Table 1.** Temperature steepness of the viscosity at $T_g$ (fragility) and of the nonergodicity factor at $T \to 0$ ($\alpha$) for several materials.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$T_g$ (K)</th>
<th>$m$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BeF$_2$</td>
<td>598</td>
<td>20</td>
<td>0.16</td>
</tr>
<tr>
<td>Silica</td>
<td>1450</td>
<td>28</td>
<td>0.19</td>
</tr>
<tr>
<td>Glycerol</td>
<td>190</td>
<td>53</td>
<td>0.32</td>
</tr>
<tr>
<td>PB$_2$</td>
<td>180</td>
<td>60</td>
<td>0.40</td>
</tr>
<tr>
<td>nBB</td>
<td>125</td>
<td>53</td>
<td>0.46</td>
</tr>
<tr>
<td>Salol</td>
<td>218</td>
<td>73</td>
<td>0.64</td>
</tr>
<tr>
<td>mtol</td>
<td>187</td>
<td>77</td>
<td>0.57</td>
</tr>
<tr>
<td>oTP</td>
<td>241</td>
<td>81</td>
<td>0.58</td>
</tr>
<tr>
<td>mTCP</td>
<td>205</td>
<td>87</td>
<td>0.59</td>
</tr>
<tr>
<td>SeI</td>
<td>308</td>
<td>87</td>
<td>0.7</td>
</tr>
</tbody>
</table>

$m$ from (26), $\alpha$ from (27). *Silica infrasil-grade sample. $m$ from (28), $\alpha$ from (29) and references therein. **Polystyrene: $m$ from (21). $\alpha$ from the experiment in (33). ††Metatricresylphosphate: $m$ from (23), $\alpha$ from unpublished data.
lines) contributions to the scattering intensity are shown, and one can appreciate in the raw data the change of relative intensity as a function of $T$. As far as the $Q$ dependence is concerned, $f(Q,T)$ follows in phase the oscillations of the static structure factor and is almost $Q$-independent in the $Q \rightarrow 0$ region, where $S(Q)$ is almost constant (22), (see the inset of Fig. 2). We focus on this small-$Q$ region. From the integrated intensities of the elastic and inelastic contributions, obtained by a fitting procedure (see eqs. S1 and S2 of the supporting on line material), the $T$ dependence of $f(Q,T)$ is obtained. The values of $f(Q,T)$ are obtained by numerical integration of the form (23).

In this work, we adopt the low-$Q$ limit, $Q \rightarrow 0$, for the inelastic contribution of the scattered intensity, $I(Q)$. This approximation, known as the Debye-Waller approximation, is valid at low $Q$ and is based on the assumption that the mean square displacement of the particles is small compared to the wavelength of the scattered light. This approximation is often used in the analysis of inelastic x-ray scattering (IXS) data to study the dynamics of condensed matter systems.

We further examine how the temperature dependence of $f(Q,T)$ in the $T \rightarrow 0$ limit, we invoke the harmonic approximation for the vibrational dynamics. This allows one to express $f(Q,T)$ in terms of the vibrational properties of the systems; that is, the eigenvalues $(\omega_i)$ and eigenvectors $(\mathbf{e}_i)$ of the potential energy Hessian, evaluated at the inherent structure. Using the harmonic approximation for $S_{\text{inc}}(Q,\omega)$, it is straightforward (23, 24) to show that Eq. 2 reduces to:

$$f(Q,T) = \left[ 1 + \frac{K_B T Q^2}{M S_{\text{ind}}(Q)} \right]^{-1} \sum_{i} \left| \frac{\mathbf{e}_i(Q)}{\omega_i} \right|^2$$  (4)

Here $M$ is the molecular mass, $K_B$ is the Boltzmann constant, and $i$ is summed over the $N$ particles and $p$ over the $3N$ normal modes. In order to pinpoint the $T$ dependence of the nonergodicity factor in the low $Q$ limit, it is convenient to rewrite Eq. 4 as:

$$f(Q \rightarrow 0, T) = \frac{1}{1 + \alpha \frac{T}{T_g}}$$  (5)

We thus define the dimensionless quantity $\alpha$, which encompasses all the microscopic details of the system, as the eigenvalues and eigenvectors of the normal modes. These are quantities that, in turn, depend on the interaction potential and on the disordered structure. This equation provides a formal way to extract the system-dependent parameter $\alpha$ from the $T$ dependence of $f(Q,T)$, derived from the IXS data. This has motivated us to revisit the large amount of IXS data available for glasses at low $T$, where the harmonic approximation, and therefore Eq. 5, is not expected to be valid. As it can be seen in Fig. 2, the observed $T$ dependence of $f(Q,T)$ is fully consistent with the functional form predicted by Eq. 5, and this allows us to determine $\alpha$ by a least-squares minimization procedure. The derived values for $\alpha$ (such as $\alpha = 0.19$ for silica, $\alpha = 0.32$ for glycerol, and $\alpha = 0.58$ for oTP) clearly indicate a trend: The more fragile a liquid, the greater the slope of $f(Q,T)$ at $T = 0$; that is, the faster the decorrelation of the density fluctuations on increasing $T$. The fitting procedure has been applied to the whole set of available glasses, and the obtained values of $\alpha$ are reported in Table 1 and Fig. 3 as a function of the independently known fragility parameter $m$. Figure 3 clearly shows the existence of a strong correlation between $m$ and $\alpha$: The higher the fragility, the higher the value of $\alpha$; that is, the faster is the $T$ dependence of the $f(Q,T)$ parameter. The existence of a strong correlation between $\alpha$ and $m$ is further emphasized by the empirical observation that the two quantities are not only correlated but (within the statistical accuracy) are actually proportional to each other, according to the relation $m = (135 \pm 10) \alpha$. In passing, we note that the two points that lie definitely below the dotted line (selenium and salol) are the ones for which fragility determined at $T_g$ doesn’t agree well with the fragilities determined at higher temperatures (7, 25).

The observed correlation is conceptually surprising. It indicates the existence of a link between the curvatures of the potential energy function at its minima (more specifically, those visited in the glassy phase) and the other properties of the potential energy function (the energy distribution of the minima, minimum-to-minimum barrier height, distribution of the saddle order and energies, etc.) controlling the diffusive processes in supercooled liquids.

We further examine how $\alpha$ emerges from the collective density-density correlation function plateau. From Eqs. 4 and 5, the microscopic expression for $\alpha$ is found to be:

$$\alpha = \frac{K_B T Q^2}{M S_{\text{ind}}(Q)} \sum_{i} \left| \frac{\mathbf{e}_i(Q)}{\omega_i} \right|^2$$  (6)

One may in principle derive a similar parameter $\alpha_\infty$ from the temperature dependence of the self-correlator plateau. In this case, $\alpha_\infty$ is related to the familiar mean square displacement $D(\omega)$. The harmonic approximation for the Debye-Waller factor $f_s(Q,T) = \exp[-W(Q,T)]$ would lead to an equation formally identical to Eq. 5, but with $\alpha$ replaced by $\alpha_\infty$. The term $\alpha_\infty$ thus represents a long-time correlation length.

Fig. 2. Values of $f(Q^*, T)$ (Eq. 2) for $Q^* = 2$ nm$^{-1}$ reported in a $T/T_g$ scale for three representative materials (solid symbols with error bars): silica ($T_g = 1450$ K), glycerol ($T_g = 190$ K), and oTP ($T_g = 241$ K). The full line is the best fit of the experimental data to Eq. 5. These fits have been used to derive the values of $\alpha$ reported in Fig. 3. The inset shows the $Q$ dependence of $f(Q^+, T)$ for silica at $T^* = 1050$ K.

Fig. 3. Correlation plot of the kinetic fragility and the $\alpha$ parameter of the nonergodicity factor (Eq. 5). The dotted line is obtained by a fit of the data to a linear equation. It corresponds to $m = 135\alpha$, and the regression coefficient is $r = 0.994$. 


Decadal Variation of the Surface Water \(\text{PCO}_2\) in the Western and Central Equatorial Pacific

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The equatorial Pacific Ocean is one of the most important yet highly variable oceanic source areas for atmospheric carbon dioxide (CO\(_2\)). Here, we used the partial pressure of CO\(_2\) (\(\text{PCO}_2\)), measured in surface waters from 1979 through early 2001, to examine the effect on the equatorial Pacific CO\(_2\) chemistry of the Pacific Decadal Oscillation phase shift, which occurred around 1988 to 1992. During the decade before the shift, the surface water \(\text{PCO}_2\) (corrected for temperature changes and atmospheric CO\(_2\) uptake) in the central and western equatorial Pacific decreased at a mean rate of about 20 μatm per decade, whereas after the shift, it increased at about 15 μatm per decade. These changes altered the CO\(_2\) sink and source flux of the equatorial Pacific significantly.

The equatorial Pacific Ocean is known to undergo significant changes on interannual time scales (e.g., El Niño and La Niña) and on decadal time scales. This area is a major source of CO\(_2\) to the atmosphere during non-El Niño periods (1–5) but near neutral during strong El Niño periods (2–5). Over decadal time scales, the North Pacific Ocean has undergone major physical and biological changes—commonly called the Pacific Decadal Oscillation (PDO)—(6–9). The most recent and major shifts of 1977 and 1988 to 1992 have been documented on the basis of extensive physical and biological information (6–9). Although the causes and effects of PDO have been investigated in recent years, its effects on CO\(_2\) chemistry have not yet been identified. Here, we describe the changes in CO\(_2\) chemistry in the Pacific equatorial waters associated with the 1988 to 1992 PDO phase shift (6–8) using the partial pressure of CO\(_2\) (\(\text{PCO}_2\)) in seawater, which is a measure of the escaping tendency of CO\(_2\) from seawater to the overlying atmosphere.

Because of the heightened interest in the El Niño–Southern Oscillation (ENSO) events and the carbon cycle, \(\text{PCO}_2\) in surface ocean waters has been measured more frequently since 1979 over the equatorial Pacific Ocean. In this study, we assembled a database containing about 100,000 \(\text{PCO}_2\) observations made in surface waters of the equatorial belt, 5°N to 5°S, between April 1979 and March 2001 (10). We classified these into three groups: (i) the Niño 3.4 area (lat 5°S to 5°N, long 170°W to 120°W) during non-El Niño periods (which include La Niña periods); (ii) the Niño 3.4 area during El Niño periods; and (iii) the western warm pool area (lat 5°N to 5°S, long 175°E to 135°E). We identified six El Niño periods in the Niño 3.4 area on the basis of sea surface temperature (SST) anomalies (11). The remaining data were considered as occurring during non-El Niño periods.

In the Niño 3.4 area, observations during non-El Niño periods were made more or less evenly throughout the year along a total of 11 meridional transects across the equator during the pre-1990 period, and 14 transects during the post-1990 period. In this area, no measurements

References and Notes
21. Some fraction of the elastic line may in principle be related to the nonpropagating temperature fluctuations. The amount of this contribution is actually ruled out by the quantity \(\gamma_1/c_0\gamma_2\) = 0 for \(\gamma_1\) close to 1, and therefore it vanishes in the \(\gamma_1\) limit according to the harmonic approximation.
36. The authors gratefully acknowledge C. A. Angell for valuable hints and suggestions, J. Dyre, S. Sastry, F. Sciortino, and S. Yannopoulos for fruitful discussions.

Supporting Online Material
www.sciencemag.org/cgi/content/full/302/5646/849/DC1

Methods

References

21 July 2003; accepted 24 September 2003